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Phase properties of a Jaynes-Cummings model with Stark shift and Kerr medium

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Abstract. Phase properties of the field interacting with a two-level atom in a lossless cavity Jaynes-Cummings model, taking into account the level shifts produced by Stark effect with an additional Kerr medium for one-mode are studied using the phase formalism of Pegg and Barnett. It is shown in particular that phase properties of the field reflect the collapse and revival phenomena. The results for the time evolution of the phase probability distribution and the phase fluctuations are obtained. The effect of Stark shift on the phase properties in both the absence and presence of a Kerr medium is analyzed. Phase localization is found for certain choice of the parameters.

PACS. 32.80.Dz Autoionization – 32.80.Rm Multiphoton ionization and excitation to highly excited states (e.g., Rydberg states)

1 Introduction

Real systems are often approximated by simple models, which can be solved exactly. One such model is the Jaynes-Cummings model (JCM), where a single two-level atom interacts with a single cavity mode [1]. Using this model the recent experiments done with Rydberg atoms in a microwave cavity are well explained [2,3]. A Kerr medium inside the cavity can be modeled by an anharmonic oscillator [4,5]. As was shown recently by Buzek and Jex [6] the Schrödinger equation for the combination of both of these models can be solved exactly in the rotating wave approximation. The JCM with intensity dependent coupling has been discussed [7]. This model is of interest because it gives rise to commensurable Rabi frequencies. The dynamical behavior of it is exactly periodic and can be compared with the standard JCM. When either Kerr or the Stark effect are taken into consideration some of the properties are affected [8]. Phase properties of the field in the JCM are examined in [9] by using the new phase formalism introduced by Pegg and Barnett [10–12]. The time behaviour of the phase density distribution presented on a polar diagram resembles a lot that of the Q-function in phase space. Namely, the interaction forces each phase state to split into two phase states rotating in opposite directions. During the period when the counterrotating distributions are well separated, the atomic inversion shows no oscillations. When the two satellite distributions overlap again, the revival of the atomic inversion occurs. Naturally, the variance of the phase carries some information about the collapses and revivals. However, in this case care must be taken because a particular choice of the reference phase, because it may influence the calculated phase properties of the state [9,13]. The detuning effects on the phase properties for one- and two-photon JCM, the long time behaviour of the second or fourth order variances and distribution function for the phase have been discussed for the cases of on-resonance and off-resonance cases [14]. Further, both one- and two-photon JCMs have been extended to include the effects of a Kerr-like medium [15].

In this paper we study the interaction between an atomic system described by a two-level atom and the quantized radiation field in the rotating wave approximation taking into account both Kerr and Stark effects. The intensity dependent Stark effect can be employed in quantum non-demolition measurements [16–18]. Kerr effects can be observed by surrounding the atom by a nonlinear medium inside a high Q-cavity [6]. We obtain the wave function of the total system at any time $t > 0$, for one-photon Jaynes-Cummings model with both Stark shift and a Kerr effect. We use Pegg-Barnett phase formalism to study the phase properties of a coherent field interacting with this system. We exhibit the phase probability distribution, the phase variance and find phase localization for certain parameters.

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2 Basic equations

We consider the Hamiltonian for the one-photon Jaynes-Cummings model with Stark shift and a Kerr-like medium. It describes the interaction of a single-mode quantized field with a two level atom *via* a one-photon process and intensity dependent Stark shift and non-linear Kerr-like medium. The effective Hamiltonian of the system in the rotating-wave approximation can be written as

$$\hat{H} = \frac{1}{2}\omega_\circ\sigma_z + \omega\hat{a}^\dagger\hat{a} + \beta\hat{a}^\dagger\hat{a} \mid e\rangle\langle e \mid + \chi\hat{a}^{\dagger 2}\hat{a}^2 + \lambda(\hat{a}\sigma_+ + \hat{a}^\dagger\sigma_-), \quad (1)$$

where $\hat{a}^\dagger$ $(\hat{a})$ is the creation (annihilation) operators for the photon of frequency ω, λ describes the coupling to the atomic system, χ denotes the coupling to the non-linear Kerr medium and β is the parameter describing the intensity-dependent Stark shift of the two levels that are due to the virtual transitions to the intermediate relay level [6–8]. When $\beta = 0$ equation (1) reduces to that of [15] The two level atom with transition frequency $\omega_\circ$ is described by the Pauli raising and (lowering) operators $\sigma_+, (\sigma_-)$ and the inversion operator σ_z. It is easy to prove that the Hamiltonian (1) has the following two constants of motion $\hat{H}_\circ$ and $\hat{H}_{in}$, where

$$\hat{H} = \hat{H}_\circ + \hat{H}_{in}, \quad (2)$$

$$\hat{H}_\circ = \omega_\circ(\sigma_z + \hat{a}^\dagger\hat{a}) \quad (3)$$

and $\hat{H}_{in}$ is the interaction Hamiltonian and is given by

$$\hat{H}_{in} = \frac{\Delta}{2}\sigma_z + \beta\hat{a}^\dagger\hat{a} \mid e\rangle\langle e \mid + \chi\hat{a}^{\dagger 2}\hat{a}^2 + \lambda(\hat{a}\sigma_+ + \hat{a}^\dagger\sigma_-) \quad (4)$$

with the detuning parameter $\Delta = \omega - \omega_\circ$. The initial state of the total atom-field system can be written as

$$\mid \psi(0)\rangle = \mid \psi(0)\rangle_f \otimes \mid \psi(0)\rangle_a = \sum_{n=0} q_n \mid n, e\rangle, \quad (5)$$

means that the atom starts in its excited state, the field is assumed to be initially in a coherent state where $q_n = e^{(-\bar{n}/2)}\frac{\alpha^n}{\sqrt{n!}}$, $\alpha = \mid \alpha \mid e^{i\phi}$ and $\bar{n} = \mid \alpha \mid^2$ is the mean photon number of the coherent field. The solution of the Schrödinger equation in the interaction picture *i.e.* the wave function of the system at any time $t > 0$ is given by

$$\mid \psi(t)\rangle = \sum_{n=0}^{\infty} q_n e^{-i\gamma_n\lambda t}\Big(A_n(t) \mid n, e\rangle + B_n(t) \mid n+1, g\rangle\Big) \quad (6)$$

where the coefficients A_n and B_n are given by the formulae

$$A_n(t) = \cos\lambda t\nu_n - \frac{i}{2\lambda}[\Delta + \beta n - 2n\chi]\frac{\sin\lambda t\nu_n}{\nu_n}, \quad (7)$$

$$B_n(t) = -i\sqrt{n+1}\frac{\sin\lambda t\nu_n}{\nu_n}, \quad (8)$$

and

$$\gamma_n = \frac{1}{2\lambda}\Big(n^2\chi + \beta n\Big), \quad (9)$$

$$\nu_n = \sqrt{\left(\frac{1}{2\lambda}[\Delta + \beta n - 2n\chi]\right)^2 + n + 1}. \quad (10)$$

With the wave function $\mid \psi(t)\rangle$ calculated, any property related to the atom or the field can be calculated. It is to be noted that when we put $\beta = 0 = \chi$, we get the results of [14], while when we put $\beta = 0$, we get the results of [15], mean-while if either β or χ are taken to be zero we get the two cases of reference [7]. In what follows we shall consider the effect of both Kerr and Stark shift on dynamical behaviour of the phase properties of the system for single photon transition.

3 Phase properties

Difficulties have been found with proper description of phase variables [19]. Recently, Pegg and Barnett have suggested a new approach using the states of well-defined phase as a starting point [10–12]. To construct a phase operator that is hermitian they restrict the state space to $(s+1)$-dimensional space Ψ spanned by the first $(s+1)$ number states. The value of s can be be made arbitrary large. Using the standard procedure [9–15], the phase probability distribution, the expectation value and the variance of the Hermitian phase operator may be obtained for the field. Expectation values are first calculated in Ψ before s is allowed to tend to infinity. The set of orthogonal phase states is defined by the following form

$$\mid \theta_m\rangle = \frac{1}{\sqrt{s+1}}\sum_{n=0}^{s} e^{in\theta_m} \mid n\rangle, \quad m = 0, 1, 2, 3, ..., s \quad (11)$$

with

$$\theta_m = \theta_\circ + \frac{2\pi m}{s+1}\cdot \quad (12)$$

These states are eigenstates of the Hermitian phase operator

$$\Phi_\theta = \sum_{m=0}^{s} \theta_m \mid \theta_m\rangle\langle\theta_m \mid . \quad (13)$$

It is apparent from equation (13) that the operator Φ_θ has eigenvalues which are restricted to the interval $[\theta_\circ, \theta_\circ + 2\pi]$ where the value of the reference phase $\theta_\circ$ is arbitrary. At $t = 0$ the cavity field mode is in a coherent state, which is a particular case of the partial physical phase state [12]. Therefore, following Pegg and Barnett we choose the reference phase $\theta_\circ$ appearing in equation (12) as

$$\theta_\circ = \Phi - \pi s/(s+1), \tag{14}$$

where Φ is an arbitrary constant. Thus the phase probability distribution is given by

$$\mid \langle \theta_m \mid \psi(t) \rangle \mid^2 = \frac{1}{s+1} \left| \sum_{n=0} \langle n \mid \psi(t) \rangle e^{-in\theta_m} \right|^2 \tag{15}$$

with the expectation value

$$\langle \Phi_\theta \rangle = \sum_{m=0}^{s} \theta_m \mid \langle \theta_m \mid \psi(t) \rangle \mid^2 \tag{16}$$

and the variance

$$\langle (\Delta\Phi_\theta)^{2r} \rangle = \sum_{m=0}^{s} (\theta_m - \langle \Phi_\theta \rangle)^{2r} \mid \langle \theta_m \mid \psi(t) \rangle \mid^2 . \tag{17}$$

In this case we use the wave function given by equation (6), and by using equation (15) we obtain the probability for value for the phase distribution. Then as s tends to infinity the summation may be transformed into an integral after making the continuum replacements [9–15] This leads to a continuous phase probability distribution in the form

$$\begin{aligned} P(\theta, t) = & \frac{1}{2\pi} \Bigg[1 + 2 \sum_{n>l} b_n b_l \\ & \times \Big(A_{n,l}(t) \cos(\theta(n-l) + (\gamma_{n,l})\lambda t) \\ & + B_{n,l}(t) \sin(\theta(n-l) + (\gamma_{n,l})\lambda t) \Big) \Bigg] \end{aligned} \tag{18}$$

where $b_n = q_n e^{in\phi}$ and

$$\begin{aligned} A_{n,l}(t) = & \cos \lambda t \nu_n \cos \lambda t \nu_l \\ & + \Bigg\{ \left(\frac{1}{2\lambda} [\Delta + \beta n - 2n\chi] \right) \left(\frac{1}{2\lambda} [\Delta + \beta l - 2l\chi] \right) \\ & + \sqrt{(n+1)(l+1)} \Bigg\} \frac{\sin \lambda t \nu_l}{\nu_l} \frac{\sin \lambda t \nu_n}{\nu_n}, \end{aligned} \tag{19}$$

$$\begin{aligned} B_{n,l}(t) = & \frac{1}{2\lambda} \Bigg([\Delta + \beta n - 2n\chi] \cos \lambda t \nu_l \frac{\sin \lambda t \nu_n}{\nu_n} \\ & - [\Delta + \beta l - 2l\chi] \cos \lambda t \nu_n \frac{\sin \lambda t \nu_l}{\nu_l} \Bigg). \end{aligned} \tag{20}$$

We can find the average value of the phase operator in the form

$$\begin{aligned} \langle \Phi_\theta \rangle = & 2 \sum_{n>l} b_n b_l \frac{(-1)^{n-l}}{n-l} \Big(A_{n,l}(t) \sin(\gamma_n - \gamma_l)\lambda t \\ & - B_{n,l}(t) \cos(\gamma_n - \gamma_l)\lambda t \Big). \end{aligned} \tag{21}$$

We now proceed to calculate the variance of the hermitian phase operator. It is described by the summations in the equation (18) which may be transformed into integrals over the variable θ, over the range -π to π, then the variance is given by

$$\begin{aligned} \langle \Delta\Phi_\theta^2 \rangle = & \frac{\pi^2}{3} + 4 \sum_{n>l} b_n b_l \frac{(-1)^{n-l}}{(n-l)^2} \Big(A_{n,l} \cos(\gamma_n - \gamma_l)\lambda t \\ & + B_{n,l} \sin(\gamma_n - \gamma_l)\lambda t \Big) - (\langle \Phi_\theta \rangle)^2. \end{aligned} \tag{22}$$

In what follows we discuss numerically some of these properties.

4 Discussion and conclusions

We have computed the phase probability distribution function, the phase expectation value and variances of the phase operator, related to a system of a 2-level atom in interaction with a single mode with Stark shift and a Kerr-like medium present. In our computations, we have taken $\bar{n} = 10$, $\phi = 0$ and $\Delta = 0$

Figure 1 shows the time evolution of the phase probability distribution $P(\theta, t)$ for $\chi/\lambda = 0$ and for various values of β/λ. When β equal to zero, it is remarked that $P(\theta, t)$ exhibits symmetric splitting as λt varies as shown in Figure 1a This is the counterrotating behaviour observed earlier [9]. When $\lambda t = 0$, $P(\theta, t)$ has a single-peak structure corresponding to the initial coherent state. The peaks are symmetric about $\theta = 0$ so that the mean phase always remains equal to zero. The time behaviour of the phase probability distribution carries some information about the collapse and revival of Rabi oscillations [20]. When the phase peaks are well separated the Rabi oscillations collapse and each time as the peaks meet (at $\theta = 0$ and/or $\pm\pi$) they produce a revival see Figure 1a. When $\beta/\lambda \neq 0$, the situation is completely changed. As shown in [15], during the propagation of a coherent field in a Kerr medium the phase distribution not only shifts but also broadens. Different features are visible in the case under consideration, due to the Stark shift but with different rates. However, as we observe from Figures 1b to 1d, one of the peaks is damped as time develops. It is seen that when $\beta/\lambda = 1$, it is almost static with very small amplitudes compared with the other peak, whose rate of shift becomes faster when plotted in a phase space as in [9,13] this would show a small blob at rest while the other larger blob moves faster around in this space. With larger β/λ (= 5 in Fig. 1d) the last peak is localized and the continuous shift observed for $\beta/\lambda \leq 1$ is lost. The irregularity for $\beta/\lambda \leq 1$ exhibits itself in the variance and mean value of the phase see Figure 3a. But as β/λ increases regular patterns in both the mean value and the variance are apparent for $\beta/\lambda = 1$ in Figure 3c and $\beta/\lambda = 5$ in Figure 3d. It is to be remarked that the phase variances assume its peak at the collapse period for the mean photon number.

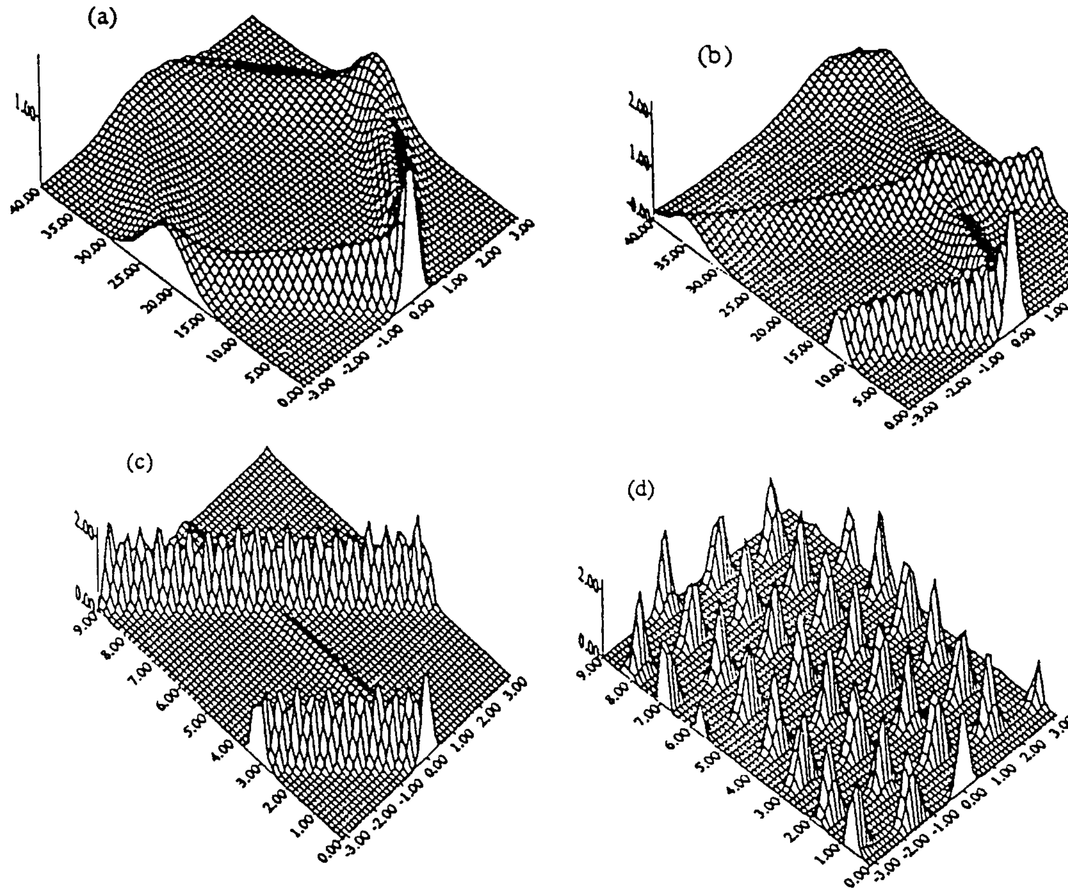

Fig. 1. Plots of the phase distribution $P(\theta, t)$ as a function of the scaled time λt for, $\bar{n} = 10$, $\chi/\lambda = 0$ and (a) $\beta/\lambda = 0$ (b) $\beta/\lambda = 0.2$ (c) $\beta/\lambda = 1$ (d) $\beta/\lambda = 5$.

Reduction in the amplitude of the right hand shifted peak can be attributed to the coefficient of the sinusoidal functions with arguments $\theta(n-l) + \gamma_{n,l}\lambda t \pm (\nu_n - \nu_l)\lambda t$.

As a matter of fact we can write $P(\theta, t)$ of (18) in the form $P(\theta, t) = P^s + P^f$ where P^f contains summations of fast oscillating terms namely $\cos[(n-l)\theta + \gamma_{n,l}\lambda t \pm (\nu_n + \nu_l)\lambda t]$. These terms do not almost contribute to the probability. The main contribution comes from the slowly oscillating terms with summation of the form $\cos[(n-l)\theta + \gamma_{n,l}\lambda t \pm (\nu_n - \nu_l)\lambda t]$. The amplitudes of these two terms in the sum are of the ratio

$$\begin{aligned}&(1 + \cos(\xi_n - \xi_l) - \sin\xi_n - \sin\xi_l) :\\ &(1 + \cos(\xi_n - \xi_l) + \sin\xi_n + \sin\xi_l),\end{aligned} \tag{23}$$

where $\cos\xi_n = \sqrt{n+1}/\nu_n$ and $\sin\xi_n = (\beta/2 - \chi)n$ and this shows the decrease of the amplitude observed in the Figures 1 and 2 for β/λ and $\chi/\lambda \neq 0$. Increasing the parameter β/λ leads to the decoupling of the system and the interaction with the atom becomes very small compared with the Stark shift. The dependence on the time is almost periodic in this case, and hence the localization of the phase as shown in Figure 1d. This can be seen also in Figures 3c and d. When we further take the Kerr effect through the parameter χ/λ, we observe that the Kerr medium leads to the diffusion of the peaks [15]. However the Stark effect persists when $\beta \gg \chi$. The increase of the parameter χ/λ adds irregularity to the mean phase, and the variance in the phases see Figure 4.

In conclusion, it is observed that the symmetry shown in the standard JCM for the phase distribution is no longer present once Stark or Kerr effect is added. The peaks are split but the two split peaks move with different rates. The one with the slower rate faces damping while the faster peak gains in the amplitude. With the increase in the Stark shift parameter β/λ, localization for the phase distribution is obtained. The Kerr effect on the other hand tends to damp and diffuse the distribution.

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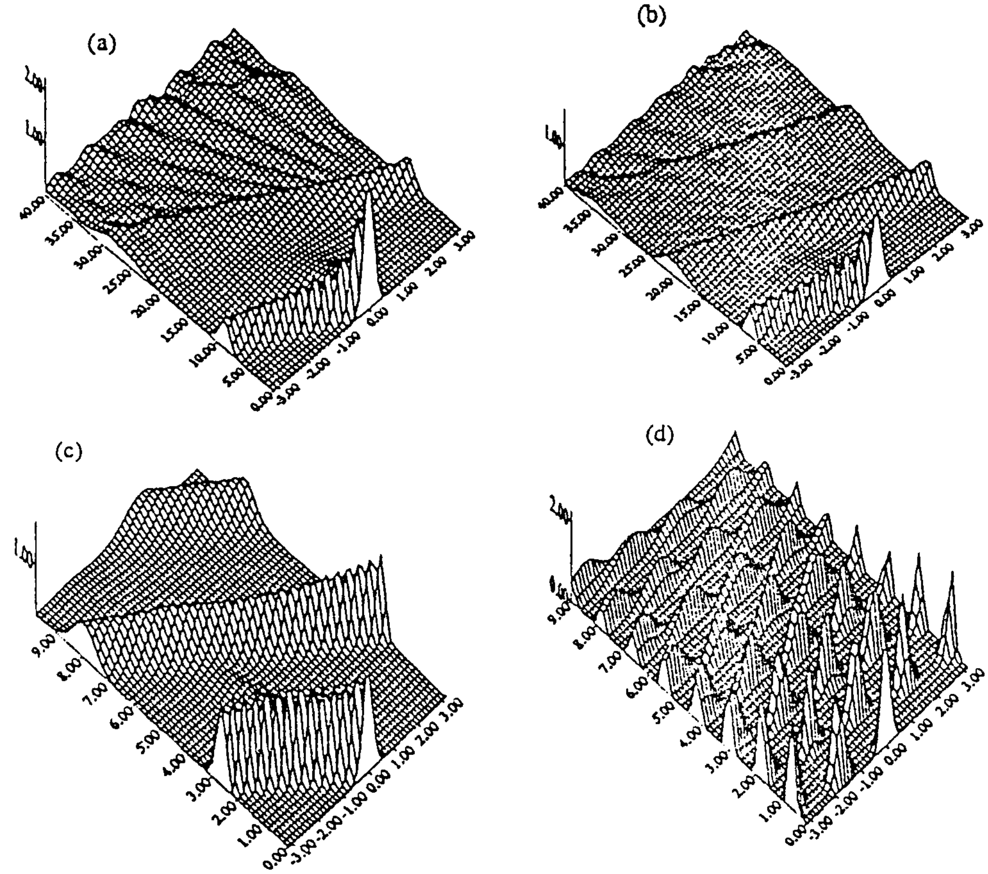

Fig. 2. The same as in Figure 1 but for $\chi/\lambda = 0.01$.

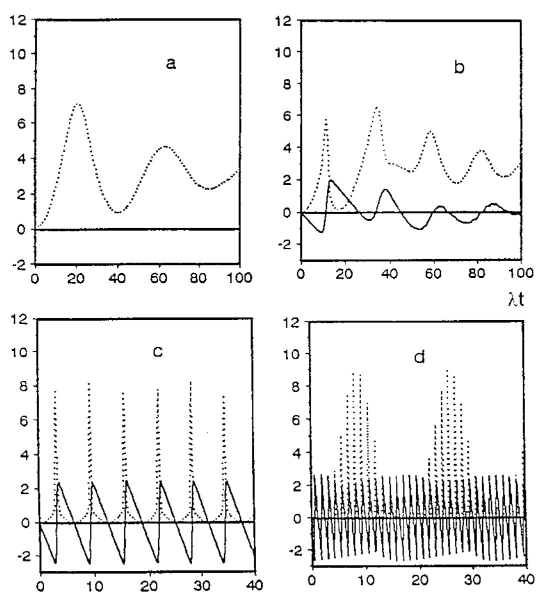

Fig. 3. Plots of phase variance $(\cdots)$ $\langle \Delta\Phi_\theta^2 \rangle$ and mean phase (——) $\langle \Phi_\theta \rangle$ as a function of the scaled time λt for, $\bar{n} = 10$, $\chi/\lambda = 0$ and (a) $\beta/\lambda = 0$, (b) $\beta/\lambda = 0.2$, (c) $\beta/\lambda = 1$, (d) $\beta/\lambda = 5$.

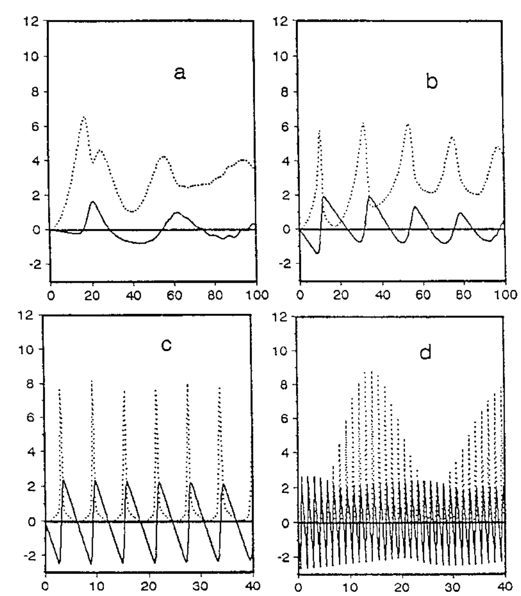

Fig. 4. The same as in Figure 3 but for $\chi/\lambda = 0.01$.

References

1. E.T. Jaynes, F.W. Cummings, Proc. IEEE, **51**, 89 (1963).
2. G. Remp, H. Walther, N. Klein, Phys. Rev. Lett. **58**, 353 (1987).
3. F. Diedrich, J. Krause, G. Rempe, M.O. Scully, H. Walter, IEEE J. Quant. Electron **24**, 1314 (1988).
4. B. Yurke, Stoler, Phys. Rev. Lett. **57**, 13 (1986).
5. G.S. Agrawal, D. Stoler, Phys. Rev. Lett. **57**, 13 (1989).
6. V. Buzek, I. Jex, Opt. Comm. **78**, 425 (1990).
7. B. Buck , C.V. Sukumar, Phys. Lett. A **81**, 132 (1981).
8. H. Moya-Cessa, S. Chavez-Cerda, J.J. Sanchez-Mondragon Phys. Lett., A **205**, 51 (1995) and references therein.
9. Ho Trung Dung, R. Tanas, A.S. Shumovsky, Opt. Comm. **79**, 462 (1990).
10. D.T. Pegg, S.M. Barnett, Euro. Phys. Lett. **6**, 483 (1988).
11. S.M. Barnett, D.T. Pegg, J. Mod. Optics **36**, 7 (1989).
12. D.T. Pegg, S.M. Barnett, Phys. Rev. A **39**, 1065 (1989).
13. Ho Trung Dung, R. Tanas, A.S. Shumovsky, J. Mod. Opt. **3**, 2069 (1991).
14. A.M. Abdel-Hafez, A.-S.F. Obada, A.H. Essawy, Ann. Phys. Fr. **20**, 47 (1995).
15. Ts Gantsong, A. Joshi, R. Tanas; Quantum Semiclass. Opt. **8**, 445 (1996).
16. S. Haroche, J.M. Raymond, Adv. At. Mol. Phys. **20**, 347 (1985).
17. M. Brune, S. Haroche V. Lefevre, J.M. Raymond, N. Zagury, Phys. Rev. Lett. **65**, 976 (1990).
18. M. Brune, S. Haroche J.M. Davidovich, N. Zagury, Phys. Rev. A **45**, 5193 (1992).
19. S.M. Barnett, D.T. Pegg, J. Phys. A **19**, 3849 (1986).
20. J.H. Eberly, N.B. Narozhny, J.J. Sanchez-Mondragon, Phys. Rev. Lett. **44**, 1323 (1980); H.-I. Yoo, Eberly, Phys. Rep. **118**, 239 (1985).